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Algorithms & Data Structures Homework 0 HS 18

Exercise 0.1 *Find the Shortest Path.*

Consider a hypothetical floor plan such that the area is organized in hexagonal cells as shown in Figure 1. We begin at the cell marked *start*, and we want to reach the cell marked *end*. We can travel from one cell to another, if the two cells are neighbouring, i.e. they share an edge. Each time we cross from one cell to a destination cell, we consider the move as a single step and the destination cell as visited. We want to find the shortest path from the start to the end, minimizing the number of steps.

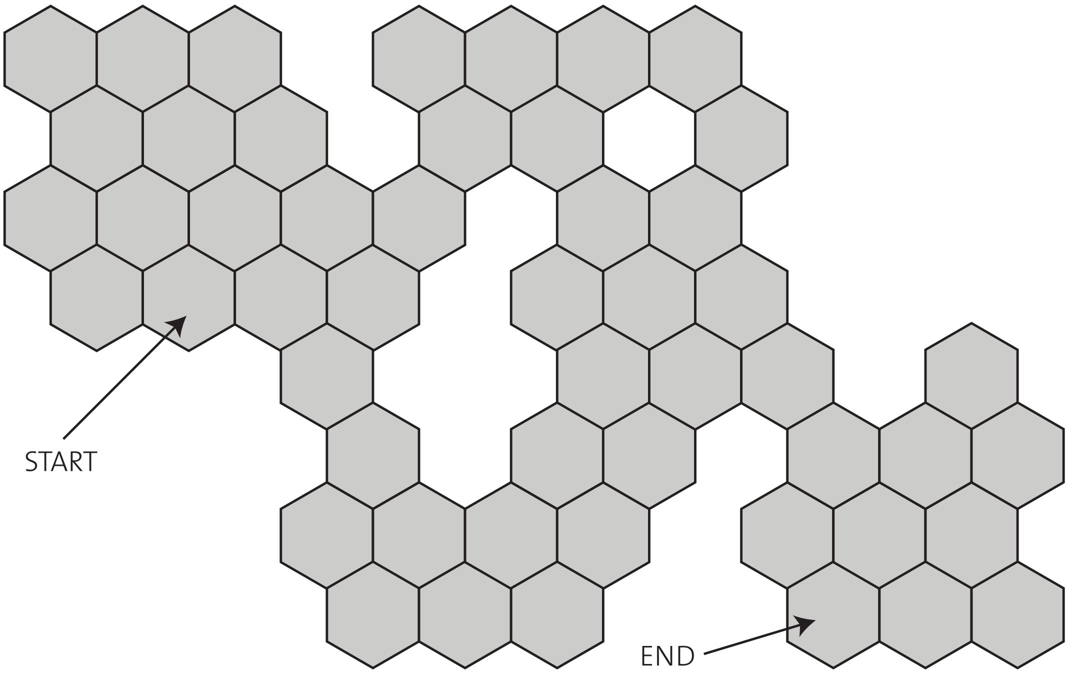

Figure 1: Floor plan

Consider the following algorithm:

1. We consider the start cell as visited, and we write the number zero on the cell.
2. If a cell has the number n written on it, we can visit all neighbouring cells, writing the number $n+1$ on it if either:
    - the neighbouring cell has not been visited before, or
    - the neighbouring cell has been visited before and holds the number m such that $m > n+1$.
3. We stop the execution of the algorithm when we can no longer execute 2.

Your tasks:

1. Execute the algorithm on the floor plan given in Figure 1, writing numbers on each cell.
2. How many steps do we need to reach the end cell?
3. For a given plan with N cells including the start and the end cell, how many steps do we need to reach the end cell, starting from the start cell, in the worst case scenario?

Exercise 0.2 *Induction.*

1. Prove via mathematical induction, that the following holds for any positive integer n:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{(n)(n+1)(2n+1)}{6}$$

   - **Base Case.**
   - **Induction Hypothesis.**
   - **Inductive Step.**

2. Prove via mathematical induction that for any positive integer n, $2n^3 + 3n^2 + n$ is divisible by 6.
   - **Base Case.**
   - **Induction Hypothesis.**
   - **Inductive Step.**

Exercise 0.3 *Coloring a Map.*

Given is a map that is divided by n (pairwise different) straight lines. You want to color the regions on the map (i.e., the areas bordered by the lines), such that no two neighbouring regions (i.e., regions that share a common segment of a line as a border) get the same color.

Prove by mathematical induction on n, that you can color every such map with 2 colors.

- **Base Case.**

- **Induction Hypothesis.**

- **Inductive Step.**

Homework: This homework does not have to be submitted and will be discussed in the first exercise session on 24.9.2018